described as global dynamic uncertainty. In the case where robust performance is sought, we may even incorporate an additive uncertainty description for the unknown higher-frequency modes, and only three uncertainty blocks will be needed, which allows an exact computation of the μ seminorm.

Acknowledgments

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Weighted-Residual Discretization for Uniform Damping and Uniform Stiffening of Structural Systems

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Introduction

LINEAR feedback control methods for full-dimensional control of structural systems are well established. One accepted approach is to control the structure's motion by controlling its modes.[1] The designer first prescribes a desirable dynamic performance for each controlled mode and then synthesizes a full-dimensional control from the modal control forces. Another approach, which is particularly robust in the presence of modeling error and actuator failure, is called decentralized control or local control.[2] Using this approach, the full-dimensional control forces and the sensor measurements are related by a decentralized (local) control algorithm. Still another approach is to employ linear optimal control theory.[3] Each approach has merit—the first approach placing highest priority on dynamic performance, the second approach placing highest priority on design simplicity and robustness, and the third approach placing highest priority on optimality.

In an attempt to satisfy the requirements of all three approaches, it was later shown that uniform damping of the structure's natural modes of vibration leads to a local control that is near globally optimal.[4] The limitation of uniform damping is that it is only capable of controlling settling time and not capable of controlling peak-overshoot and steady-state error. This Note extends the uniform damping results given in Ref. 4, first showing that uniform damping and uniform stiffening of the structure's natural modes of vibration leads to a local control. The three settling-time, peak-overshoot, and steady-state error requirements are satisfied as well.

The development of the uniform damping and uniform stiffening control algorithm proceeds by first considering distributed control forces. The distributed control forces are then discretized in order to realize the uniform damping and uniform stiffening by means of discrete forces. The method of discretizing the controls that is developed in this Note is a weighted residual method. The method is capable of turning local distributed control forces into either local discrete control forces or into global discrete control forces, depending on the admissible functions used in the discretization. A numerical example shows the discretization of local distributed forces into global discrete forces.

Modal Control

The vibration of a normal-mode structural system is governed by the linear differential equation

$$\rho(\boldsymbol{P})\frac{\mathrm{d}^2\boldsymbol{u}(\boldsymbol{P},t)}{\mathrm{d}t^2}+\boldsymbol{L}\boldsymbol{u}(\boldsymbol{P},t)=\boldsymbol{f}_C(\boldsymbol{P},t)+\boldsymbol{f}_D(\boldsymbol{P},t) \tag{1}$$

where $\rho(\boldsymbol{P})$ denotes mass density at point $\boldsymbol{P}$ in the domain D of the structural system, $\boldsymbol{u}(\boldsymbol{P},t)$ denotes displacement at point P and time t, $\boldsymbol{L}$ is a self-adjoint linear operator expressing structural stiffness, $\boldsymbol{f}_C(\boldsymbol{P},t)$ is a control force, and $\boldsymbol{f}_D(\boldsymbol{P},t)$ is a quasistatic external disturbance.[5] The linear feedback control force has the general form

$$\boldsymbol{f}_C(\boldsymbol{P},t)=-\boldsymbol{G}\boldsymbol{u}(\boldsymbol{P},t)-\boldsymbol{H}\frac{\mathrm{d}\boldsymbol{u}(\boldsymbol{P},t)}{\mathrm{d}t}-\boldsymbol{I}\int\boldsymbol{u}(\boldsymbol{P},t)\,\mathrm{d}t \tag{2}$$

where $\boldsymbol{G}$, $\boldsymbol{H}$, and $\boldsymbol{I}$ denote proportional feedback, derivative feedback, and integral feedback linear operators, respectively. The structural system exhibits normal-mode behavior. Accordingly, the displacement $\boldsymbol{u}(\boldsymbol{P},t)$ is expressed as an infinite sum of natural modes of vibration $\phi_s(\boldsymbol{P})$ multiplied by modal displacements $\eta_s(t)$, as

$$\boldsymbol{u}(\boldsymbol{P},t)=\sum_{s=1}^{\infty}\phi_s(\boldsymbol{P})\eta_s(t)$$

Substituting this into Eq. (1), multiplying the result by

$$\int_D\phi_r(\boldsymbol{P})\cdot()\,\mathrm{d}D$$

and invoking the orthonormality conditions

$$\int_D\rho(\boldsymbol{P})\phi_r(\boldsymbol{P})\cdot\phi_s(\boldsymbol{P})\,\mathrm{d}D=\delta_{rs}$$

$$\int_D\boldsymbol{L}\phi_r(\boldsymbol{P})\cdot\phi_s(\boldsymbol{P})\,\mathrm{d}D=\omega_r^2\delta_{rs},\qquad(r,s=1,2,\ldots)$$

we get the modal equations of motion

$$\frac{\mathrm{d}^2\eta_r(t)}{\mathrm{d}t^2}+\eta_r(t)=N_{\mathrm{C}r}(t)+N_{\mathrm{D}r} \tag{3a}$$

where

$$N_{\mathrm{C}r}(t)=\int_D\phi_r(\boldsymbol{P})\cdot\boldsymbol{f}_C(\boldsymbol{P},t)\,\mathrm{d}D \tag{3b}$$

$$N_{\mathrm{D}r}=\int_D\phi_r(\boldsymbol{P})\cdot\boldsymbol{f}_D(\boldsymbol{P},t)\,\mathrm{d}D \tag{3c}$$

denote modal control forces and modal disturbance forces, respectively. Next, substitute Eq. (2) into Eq. (3b) to yield the modal control algorithm

$$N_{\mathrm{C}r}(t)=-\sum_{s=1}^{\infty}\left[g_{rs}\eta_s(t)+h_{rs}\frac{\mathrm{d}\eta_s(t)}{\mathrm{d}t}+i_{rs}\int\eta_s(t)\,\mathrm{d}t\right] \tag{4}$$

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where

$$g_{rs} = \int_D \phi_r(\boldsymbol{P}) \cdot \boldsymbol{G}\phi_s(\boldsymbol{P})\,\mathrm{d}D, \qquad h_{rs} = \int_D \phi_r(\boldsymbol{P}) \cdot \boldsymbol{H}\phi_s(\boldsymbol{P})\,\mathrm{d}D$$

$$i_{rs} = \int_D \phi_r(\boldsymbol{P}) \cdot \boldsymbol{I}\phi_s(\boldsymbol{P})\,\mathrm{d}D$$

denote modal control gains. In full-dimensional modal control it is common practice to control the modes independent of one another—taking modal gains in the form $g_{rs} = g_r\delta_{rs}$, $h_{rs} = h_r\delta_{rs}$, and $i_{rs} = i_r\delta_{rs}$. It is a simple matter to verify that the control gain operators $\boldsymbol{G}$, $\boldsymbol{H}$, and $\boldsymbol{I}$ are then of the general self-adjoint form. Using the identity operator expressed as $\boldsymbol{1}(\,) = \Sigma\phi_r(\boldsymbol{P}) \int_D \rho(\boldsymbol{P})\phi_r(\boldsymbol{P}) \cdot (\,)\,\mathrm{d}D$, we find

$$\boldsymbol{G}(\,) = \rho(\boldsymbol{P}) \sum_{r=1}^{\infty} g_r\phi_r(\boldsymbol{P}) \int_D \rho(\boldsymbol{P})\phi_r(\boldsymbol{P}) \cdot (\,)\,\mathrm{d}D \tag{5a}$$

$$\boldsymbol{H}(\,) = \rho(\boldsymbol{P}) \sum_{r=1}^{\infty} h_r\phi_r(\boldsymbol{P}) \int_D \rho(\boldsymbol{P})\phi_r(\boldsymbol{P}) \cdot (\,)\,\mathrm{d}D \tag{5b}$$

$$\boldsymbol{I}(\,) = \rho(\boldsymbol{P}) \sum_{r=1}^{\infty} i_r\phi_r(\boldsymbol{P}) \int_D \rho(\boldsymbol{P})\phi_r(\boldsymbol{P}) \cdot (\,)\,\mathrm{d}D \tag{5c}$$

Substituting Eq. (4) into Eq. (3a) yields the modal equations governing the closed-loop motion of the normal-mode structure. Without loss of generality, assume that the modal displacement $\eta_r(t)$ is underdamped, in which case $\eta_r(t)$ is given by $\eta_r(t) = e_r^{-\alpha t}[A_r\cos(\beta_r t) + B_r\sin(\beta_r t)] + C_r e_r^{-\gamma t}$, where α_r denotes the rth-controlled modal damping rate of the vibration, β_r denotes the rth-controlled modal frequency, γ_r denotes the rth modal damping rate of the steady-state error, and where A_r, B_r, and C_r are constants that depend on the initial conditions. In the absence of feedback, $\alpha_r = \gamma_r = 0$ and $\beta_r = \omega_r$. In the presence of feedback, the modal control gains are related to the dynamic performance parameters by

$$g_r = \alpha_r^2 + \beta_r^2 + 2\alpha_r\gamma_r - \omega_r^2, \qquad h_r = 2\alpha_r + \gamma_r$$
$$i_r = \gamma_r\left(\alpha_r^2 + \beta_r^2\right) \tag{6}$$

Closed-form expressions for the constants A_r, B_r, and C_r are given in Ref. 6.

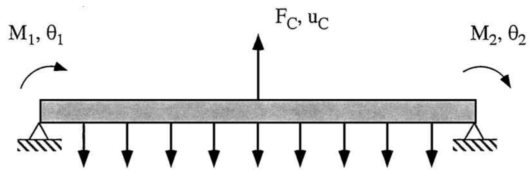
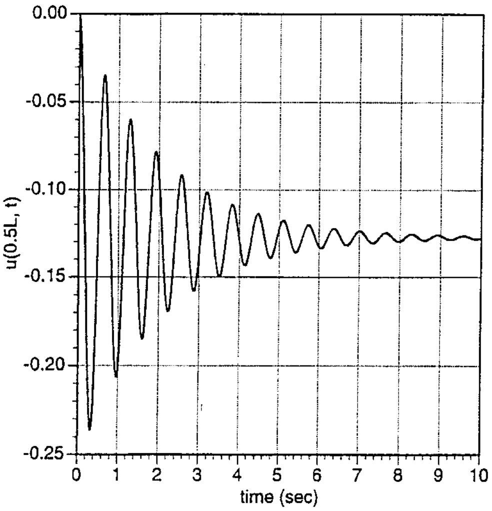
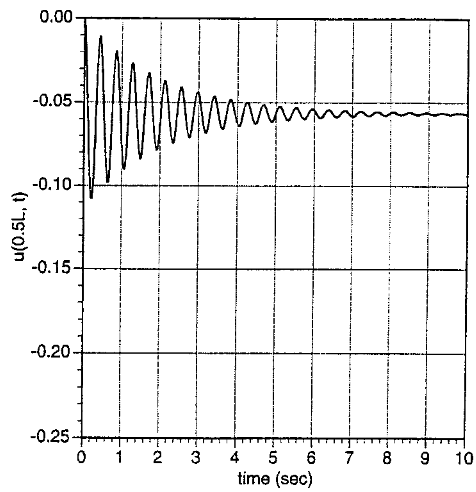


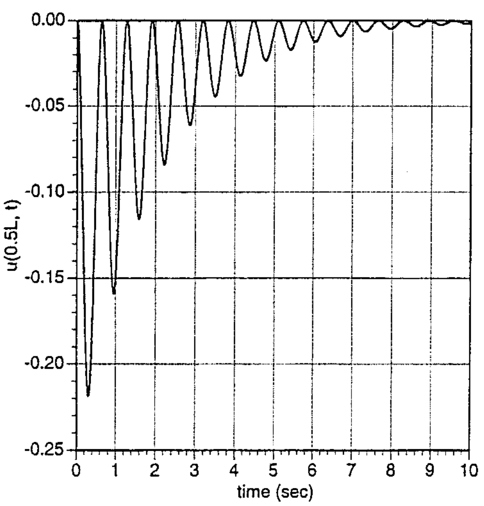
Fig. 1 Numerical example.



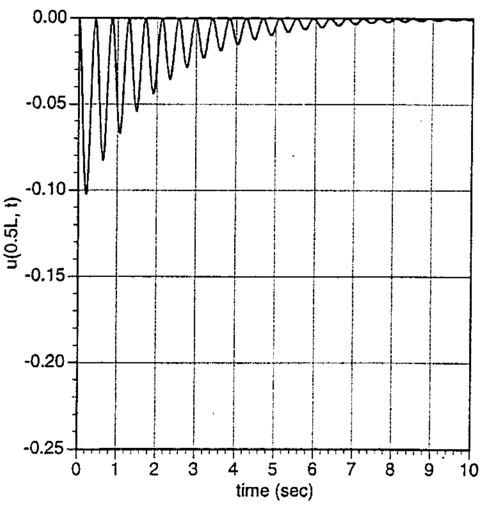
Uniform vibration damping (γ = 0.0, ξ = 1.0, α = 0.5)



Uniform vibration damping and stiffening (γ = 0.0, ξ = 1.5, α = 0.5)



Uniform vibration/bias damping (γ = 0.5, ξ = 1.0, α = 0.5)



Uniform vibration/bias damping and stiffening (γ = 0.5, ξ = 1.5, α = 0.5)

Fig. 2 Distributed control.

Uniform Damping and Uniform Stiffening: Distributed Control

The control law in Eqs. (2) and (4), with $g_{rs} = g_r\delta_{rs}$, $h_{rs} = h_r\delta_{rs}$, and $i_{rs} = i_r\delta_{rs}$, is in a form that is known as independent modal-space control or natural control (Refs. 1, 6, and 7). Notice that the control force at any point $\boldsymbol{P}$ is determined from measurements that are taken over the entire domain of the structure and that a predictive model of the structure is required. In particular the modal quantities ϕ_r, ω_r, and ρ are needed.

Although a predictive model is required, modeling errors cannot destabilize the system under certain conditions. To see this, the energy of the closed-loop system

$$E(t) = \frac{1}{2}\int_D \rho(\boldsymbol{P})\,\frac{\mathrm{d}\boldsymbol{u}(\boldsymbol{P},t)}{\mathrm{d}t}\cdot\frac{\mathrm{d}\boldsymbol{u}(\boldsymbol{P},t)}{\mathrm{d}t}\,\mathrm{d}D + \frac{1}{2}\int_D \boldsymbol{u}(\boldsymbol{P},t)\cdot(\boldsymbol{L}+\boldsymbol{G})\boldsymbol{u}(\boldsymbol{P},t)\,\mathrm{d}D$$

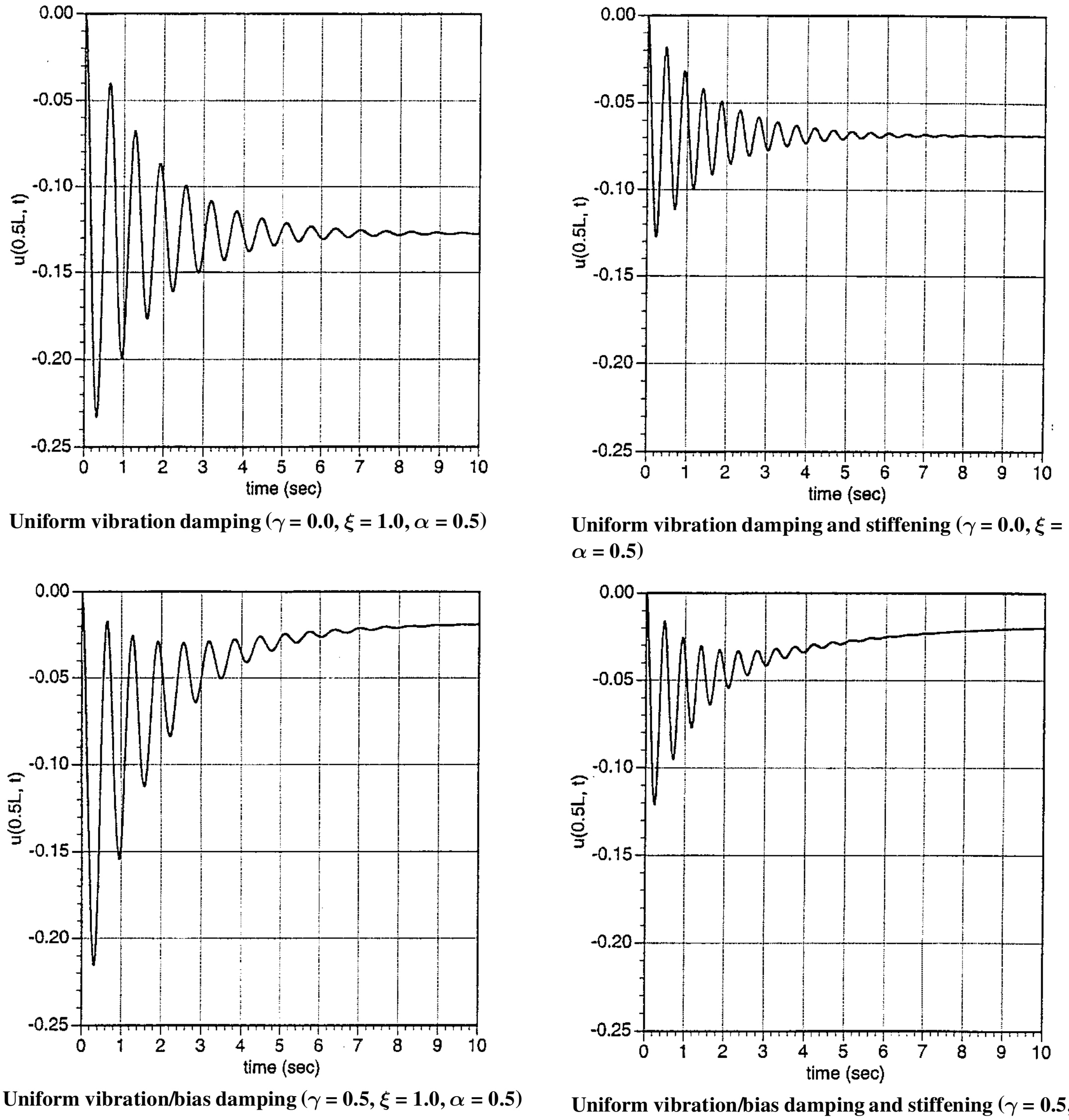
is differentiated with respect to time to get

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = -\int_D \frac{\mathrm{d}\boldsymbol{u}(\boldsymbol{P},t)}{\mathrm{d}t}\cdot\boldsymbol{H}\,\frac{\mathrm{d}\boldsymbol{u}(\boldsymbol{P},t)}{\mathrm{d}t}\,\mathrm{d}D - \int_D \frac{\mathrm{d}\boldsymbol{u}(\boldsymbol{P},t)}{\mathrm{d}t}\cdot\boldsymbol{I}\left[\int \boldsymbol{u}(\boldsymbol{P},t)\,\mathrm{d}t\right]\mathrm{d}D$$

In the absence of integral feedback ($\boldsymbol{I} = 0$), the velocity feedback operator $\boldsymbol{H}$ is positive semidefinite regardless of the physical parameters, implying that $\mathrm{d}E(t)/\mathrm{d}t$ is negative. However, in the presence of integral feedback, stability depends on the physical parameters of the system. For example, in a single degree-of-freedom mass-spring system subject to feedback, it is easy to show by the Routh–Hurwitz criterion that stability is guaranteed if the parameters of the system are such that $0 < i < h(k+g)/m$, where k is stiffness, m is mass, g is displacement feedback gain, h is velocity feedback gain, and i is integral feedback gain. Of course, whether stability is guaranteed or not, modeling error degrades closed-loop performance through increased oscillations.

Let us now impose several additional requirements on the performance of the system in order to simplify the form of the control algorithm. Let $\alpha_r = \alpha$, $\beta_r = \sigma\omega_r$, and $\gamma_r = \gamma$, where α denotes the uniform damping rate of the vibration, σ is a uniform stiffening parameter, and γ denotes the uniform damping rate of the steady-state error. Substituting this into Eqs. (2), (5), and (6) yields

$$\begin{aligned} f_C(\boldsymbol{P},t) = &-\sum_{r=1}^{\infty}\left(\alpha_r^2+\beta_r^2+2\alpha_r\gamma_r-\omega_r^2\right)\phi_r(\boldsymbol{P}) \\ &\times\int_D \rho(\boldsymbol{P})\phi_r(\boldsymbol{P})\cdot\boldsymbol{u}(\boldsymbol{P},t)\,\mathrm{d}D - \sum_{r=1}^{\infty}(2\alpha_r+\gamma_r)_r\phi_r(\boldsymbol{P}) \\ &\times\int_D \rho(\boldsymbol{P})\phi_r(\boldsymbol{P})\cdot\frac{\mathrm{d}\boldsymbol{u}(\boldsymbol{P},t)}{\mathrm{d}t}\,\mathrm{d}D - \sum_{r=1}^{\infty}\left[\gamma_r\left(\alpha_r^2+\beta_r^2\right)\right]\phi_r(\boldsymbol{P}) \\ &\times\int_D \rho(\boldsymbol{P})\phi_r(\boldsymbol{P})\cdot\int \boldsymbol{u}(\boldsymbol{P},t)\,\mathrm{d}t\,\mathrm{d}D \end{aligned} \tag{7}$$



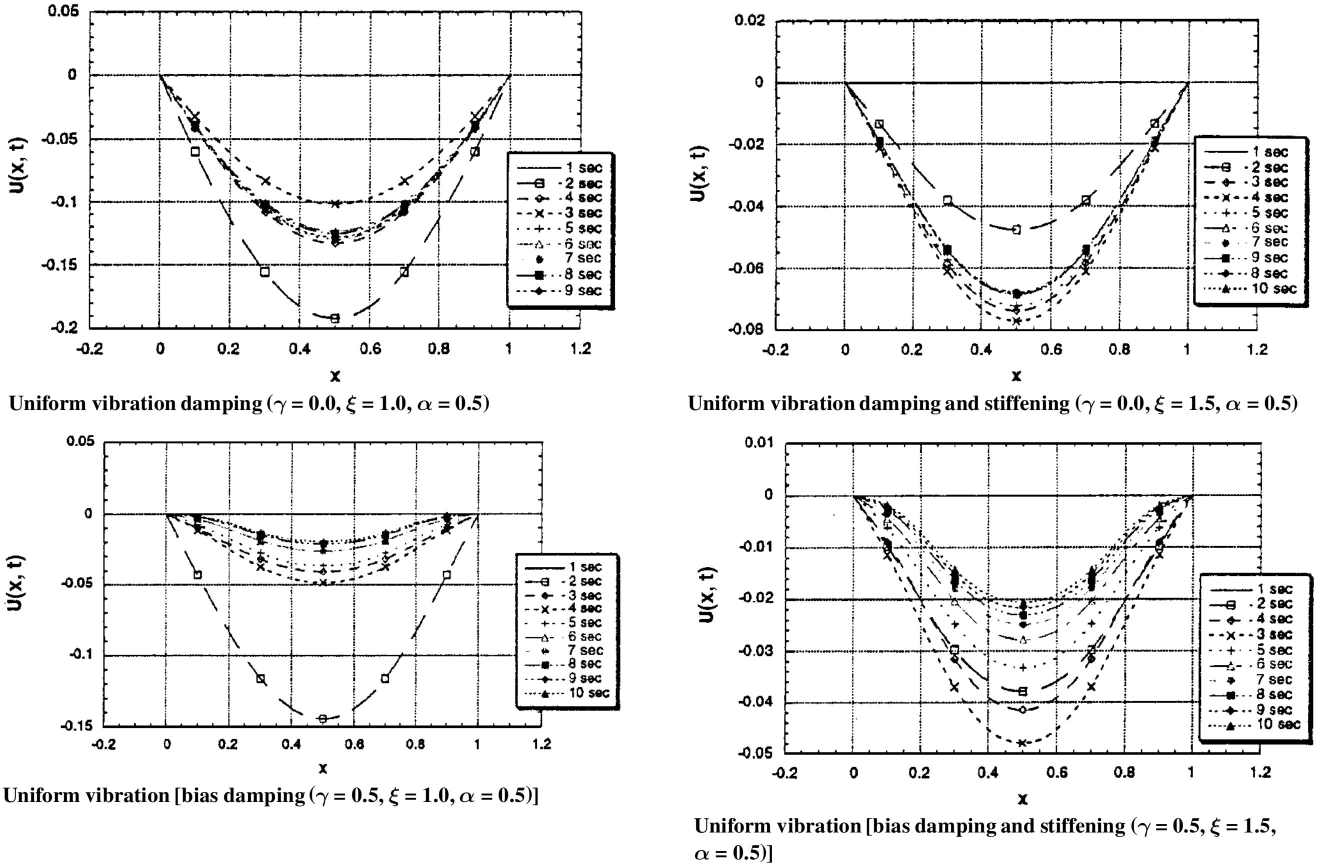
Uniform vibration damping (γ = 0.0, ξ = 1.0, α = 0.5)

Uniform vibration damping and stiffening (γ = 0.0, ξ = 1.5, α = 0.5)

Uniform vibration/bias damping (γ = 0.5, ξ = 1.0, α = 0.5)

Uniform vibration/bias damping and stiffening (γ = 0.5, ξ = 1.5, α = 0.5)

Fig. 3 Discrete control.



Uniform vibration damping (γ = 0.0, ξ = 1.0, α = 0.5)

Uniform vibration damping and stiffening (γ = 0.0, ξ = 1.5, α = 0.5)

Uniform vibration [bias damping (γ = 0.5, ξ = 1.0, α = 0.5)]

Uniform vibration [bias damping and stiffening (γ = 0.5, ξ = 1.5, α = 0.5)]

Fig. 4 Discrete control (time-lapse plots).

Invoking the orthonormality conditions, the general form of the uniform damping and uniform stiffening control force simplifies to

$$f_C(\boldsymbol{P}, t) = -[(\alpha^2 + 2\alpha\gamma)\rho(\boldsymbol{P}) + (\sigma^2 - 1)\boldsymbol{L}]\boldsymbol{u}(\boldsymbol{P}, t) - [(2\alpha + \gamma)\rho(\boldsymbol{P})]\frac{\mathrm{d}u(\boldsymbol{P}, t)}{\mathrm{d}t} - \gamma[\alpha^2\rho(\boldsymbol{P}) + \sigma^2\boldsymbol{L}]\int \boldsymbol{u}(\boldsymbol{P}, t)\,\mathrm{d}t \tag{8}$$

[Equation (8) is found using the stiffness operator expressed as $\boldsymbol{L}() = \rho(\boldsymbol{P})\Sigma\omega_r^2\phi_r(\boldsymbol{P})\int_D \rho(\boldsymbol{P})\phi_r(\boldsymbol{P})\cdot()\,\mathrm{d}D$].

Notice that the control force $f_C(\boldsymbol{P}, t)$ at point $\boldsymbol{P}$ now depends only on measurements located at point $\boldsymbol{P}$. Furthermore, observe that the modal quantities in Eq. (7), ϕ_r and ω_r, have been absorbed into the physical parameters $\rho(\boldsymbol{P})$ and $\boldsymbol{L}$ that now appear in Eq. (8). This is summarized in the principle that follows.

Uniform Damping and Uniform Stiffening Principle: The uniform damping and uniform stiffening control algorithm is fully *localized*; the distributed control force depends only on local measurements and on local physical parameters ρ and $\boldsymbol{L}$.

Uniform Damping and Uniform Stiffening: Discrete Control

The uniform damping and uniform stiffening control algorithm, Eq. (8), is distributed, which means that the control force density is distributed over the domain of the structural system. To realize uniform damping and uniform stiffening by means of discrete forces, an approach to discretizing the distributed forces is necessary. One approach is to discretize the controls in such a way that the "localized" nature of the control algorithm is preserved during the discretization process. This approach was adopted in Refs. 4 and 8. In this section, a general approach to discretizing the control algorithm is described. The approach is to minimize a weighted residual associated with the control algorithm. To accomplish this, define a set of N admissible functions $\varphi_r(\boldsymbol{P})$, $(r = 1, 2, \ldots, N)$ and express the displacement as

$$\boldsymbol{u}(\boldsymbol{P}, t) = \sum_{r=1}^{N} \varphi_r(\boldsymbol{P})U_r(t) \tag{9}$$

where N is equal to the number of discrete control forces. The discrete control forces are found by setting to zero the projection of the error in the direction of the admissible functions (the weighted residuals). Denoting the discrete control force by $f_{\mathrm{CD}}(\boldsymbol{P}, t)$ (represented as a distributed force), the projections are

$$0 = \int_D \varphi_r(\boldsymbol{P}) \cdot [f_C(\boldsymbol{P}, t) - f_{\mathrm{CD}}(\boldsymbol{P}, t)]\,\mathrm{d}D, \qquad (r = 1, 2, \ldots, N) \tag{10}$$

Substituting Eqs. (8) and (9) into Eq. (10) yields the discrete uniform damping and uniform stiffening control algorithm

$$\boldsymbol{F}(t) = -\boldsymbol{G}_D\boldsymbol{U}(t) - \boldsymbol{H}_D\frac{\mathrm{d}\boldsymbol{U}(t)}{\mathrm{d}t} - \boldsymbol{I}_D\int \boldsymbol{U}(t)\,\mathrm{d}t \tag{11}$$

where $\boldsymbol{F}(t) = [F_1(t)F_2(t)\cdots F_N(t)]^T$ is an N-dimensional vector of generalized control forces, in which

$$\boldsymbol{F}_r(t) = \int_D \varphi_r(\boldsymbol{P}) \cdot f_{\mathrm{CD}}(\boldsymbol{P}, t)\,\mathrm{d}D$$

and $\boldsymbol{U}(t) = [U_1(t)U_2(t)\cdots U_N(t)]^T$ is an N-dimensional vector of generalized displacement measurements. The entries of the $N \times N$ generalized control gain matrices $\boldsymbol{G}_D$, $\boldsymbol{H}_D$, and $\boldsymbol{I}_D$ in Eq. (11) are

$$G_{Drs} = \int_D \varphi_r(\boldsymbol{P}) \cdot \boldsymbol{G}\varphi_s(\boldsymbol{P})\,\mathrm{d}D \tag{12a}$$

$$H_{Drs} = \int_D \varphi_r(\boldsymbol{P}) \cdot H\varphi_s(\boldsymbol{P})\,\mathrm{d}D \tag{12b}$$

$$I_{Drs} = \int_D \varphi_r(\boldsymbol{P}) \cdot I\varphi_s(\boldsymbol{P})\,\mathrm{d}D \tag{12c}$$

for the $\boldsymbol{G}$, $\boldsymbol{H}$, and $\boldsymbol{I}$ given in Eq. (8). The selection of admissible functions is critical to the discretization of the control algorithm.

As in finite element analysis, depending on the admissible functions selected, the generalized coordinates become discrete coordinates, like nodal displacements, nodal slopes, or nodal strains. (In finite element analysis, the discretized equations governing the motion of the structure are obtained by minimizing a weighted residual. Depending on the admissible functions selected, the discretized coordinates become nodal displacements, nodal angles, etc.) The numerical illustration given in the next section clarifies this point.

Numerical Illustration

Consider a simple pinned-pinned beam of length L undergoing bending vibration. The uniform bending operator is $L() = EI\,\mathrm{d}^4()/\mathrm{d}x^4$, and the beam's associated natural modes of vibration and natural frequencies of oscillation are given by $\phi_r(x) = (2/\rho L)^{1/2}\sin(r\pi x/L)$ and $\omega_r = r\pi(EI/\rho L^4)^{1/2}$, respectively. Assume that the beam is initially at rest and undeformed when a uniform gravity load is applied. For simulation purposes we assumed that 20 modes participate significantly in the overall system response.

The beam shall be controlled two ways: using distributed uniform damping and stiffening and using discrete uniform damping and stiffening (see Fig. 1). In the discrete case we shall let uniform damping and stiffening be realized using two discrete moments $F_1(t) = M_1(t)$ and $F_2(t) = M_2(t)$ at each end of the beam and using a discrete force $F_3(t) = F_C(t)$ in the center of the beam. The associated discrete measurements are two angular displacement measurements $U_1(t) = \theta_1(t)$ and $U_2(t) = \theta_2(t)$ at each end of the beam and a displacement measurement $U_3(t) = U_C(t)$ in the center of the beam. The admissible functions $\varphi_r(x)$, $(r = 1, 2, 3)$ that turn the generalized forces and measurements into these discrete forces and measurements are determined from Eq. (9) to be

$$\varphi_1(x) = L\left[-\tfrac{3}{8} + (x/L) - \tfrac{1}{2}(x/L)^2\right]$$

$$\varphi_2(x) = L\left[-\tfrac{1}{8} + \tfrac{1}{2}(x/L)^2\right], \qquad \varphi_3(x) = 1$$

The discrete control forces are expressed as a distributed force as

$$f_{\mathrm{CD}}(x,t) = -M_1(t)\frac{\mathrm{d}[\delta(x-0)]}{\mathrm{d}x} - M_2(t)\frac{\mathrm{d}[\delta(x-L)]}{\mathrm{d}x} + F_C(t)\delta\left(\frac{x-L}{2}\right)$$

The displacements in the center of the beam using distributed uniform damping and uniform stiffening are shown in Fig. 2. The displacements in the center of the beam and the associated time-lapse plots using discrete uniform damping and uniform stiffening are shown in Figs. 3 and 4. Comparing the distributed control with the discrete control, notice that the discretization errors associated with vibration damping and uniform stiffening are indistinguishable, whereas the discretization error associated with the damping of the steady-state error is significant. The discretization error associated with the steady-state error is significant because the steady-state error is not completely controllable (cannot be completely removed) regardless of the control gains.

Conclusions

This Note extends the uniform vibration damping results of Ref. 4 in two directions. First, uniform damping of the steady-state error and uniform stiffening of the structural vibration together with the uniform damping of the vibration were shown to lead to a localized control algorithm, like uniform vibration damping alone. This result was articulated in a Uniform Damping and Uniform Stiffening Principle stating that the associate "control algorithm is fully localized—the control force depends only on local measurements and on local physical parameters (ρ and $\boldsymbol{L}$)." Secondly, a general weighted-residual method was developed for the discretization of the distributed forces into discrete forces. The method was illustrated in a numerical example.

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Horizontal Control Effector Sizing for Supersonic Transport Aircraft

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I. Introduction

THE use of an automatic flight augmentation system is commonplace on a modern aircraft. Its benefits include alleviation of undesirable flight characteristics, reduction of pilot workload, and increase in performance and fuel efficiency. Therefore, feedback (dynamic) considerations should be included in determining the sizes of aircraft control surfaces. Traditionally, only static constraints have been used for control surface sizing. For example, in the case of a horizontal tail of a given volume, constraints are calculated that limit the fore and aft travel of the c.g. Constraints that limit the forward c.g. position include 1) sufficient nose-up pitch acceleration at the rotation speed (nose-wheel lift off) and 2) sufficient nose-up pitch acceleration at the approach speed in the landing configuration (go-around). Constraints that limit aft c.g. position include 1) at brake release with maximum thrust sufficient weight on the nose gear (tip back), 2) pitch-up acceleration at the rotation speed (nose-wheel lift off), and 3) sufficient nose-down pitch acceleration at minimum flying speeds.[1] However, for the aft c.g. locations at the approach flight condition of a supersonic transport aircraft, dynamic constraints may be more restrictive than the static ones.

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